

Math 2167: Homework 4

Due: Friday, September 20th

Two more questions adapted from Davis, Porta, and Uhl's fantastic book *Calculus&Mathematica: Derivatives, Measuring Growth*.

You've started working for a grocery store selling *Bachelor Chow*, the premium human-kibble. You sell B bags of *Bachelor Chow* per year. Your boss has the following question for you:

How many bags per order should be requested to minimize costs?

We're going to help you out here. Let x be the number of bags requested per order. The cost of making an order is

$$F + Px$$

where F is a fixed cost per order and accounts for the secretarial time, internet, and so on; and P is the cost for each bag, including cost of the bag, shipping, and so on. If your store needs B bags per year, there will be B/x orders, so the total ordering cost is

$$(F + Px) \cdot \frac{B}{x}$$

However, there is also storage cost. If x bags are ordered per request, then the average number of bags in inventory is $x/2$, so the storage cost is

$$S \cdot \frac{x}{2}.$$

This means that the total cost with respect to x (the number of bags requested per order) is

$$S \cdot \frac{x}{2} + (F + Px) \cdot \frac{B}{x}.$$

1) Now for the questions:

- (a) Simplify $C(x)$ so that you can use the derivative rules we know from class to compute $C'(x)$.
- (b) From your work above explain why it is true that if currently

$$0 < x < \sqrt{\frac{2FB}{S}}$$

then increasing the number of bags requested per order will lower costs.

- (c) From your work above explain why it is true that if currently

$$\sqrt{\frac{2FB}{S}} < x$$

then decreasing the number of bags requested per order will lower costs.

- (d) You decide to tell your boss that as a rule of thumb, you should set x so that

$$\frac{Sx}{2} \approx \frac{FB}{x}.$$

Explain why this is a good idea.

2) Here are some facts about semi-trucks and a trip between Chicago and New Orleans.

- (a) The trip is 750 miles.
- (b) Running at 50 mph, the truck gets around 4 miles per gallon.
- (c) For each mph increase in speed, the truck losses 1/10 of a mile per gallon in mileage.
- (d) The driver team gets \$27 per hour.
- (e) Keeping the truck on the road costs 15 dollars per hour over and above the cost of fuel.
- (f) Diesel fuel costs \$3.90 per gallon.

Write a function for the total cost of driving the truck from Chicago to New Orleans with respect to the average speed x . Use calculus to minimize costs. Explain your reasoning.