Math 2167: Homework 5
Due: Friday, October 11th

1) You find that you are dealing with a disease with

$$\text{infect}'(x) = 0.000004 \cdot \text{suscept}(x) \cdot \text{infect}(x) - 0.047 \cdot \text{infect}(x),$$
$$\text{suscept}'(x) = -0.000004 \cdot \text{suscept}(x) \cdot \text{infect}(x).$$

Where infect(x) is the number of infected people on day x, and suscept(x) is the number of susceptible people on day x.

(a) Explain why the two formulas above are reasonable, though you don't need to worry about the precise values of the coefficients.

(b) Discuss the outlook on day 13 if you know suscept(13) = 5028 and infect(13) = 4012. In particular, you should talk about both infect$'(x)$ and suscept$'(x)$.

(c) Compute infect$''(x)$ on day 13. What does this say about the outlook?

2) One day you teach your middle school students about the area and circumference of a circle and you write the following formulas on the board:

$$A = \pi r^2 \qquad \text{and} \qquad C = 2\pi r.$$

The next day you add the formulas for the volume and surface-area of a sphere:

$$V = \frac{4}{3}\pi r^3 \qquad \text{and} \qquad S = 4\pi r^2.$$

"That's odd," you think, it looks like

$$\frac{dA}{dr} = C \qquad \text{and} \qquad \frac{dV}{dr} = S.$$

You are right.

(a) Explain, drawing pictures as necessary, why you are right. Big hint: Think about how $f'(x)h \approx f(x+h) - f(x)$ when h is near zero.

(b) After solving the problem above, you think about the volume and surface area of a cube:

$$V = a^3 \qquad \text{and} \qquad S = 6a^2.$$

This bothers you—following the pattern from your work on circles and spheres, shouldn't $S = \frac{dV}{da}$? Explain why or why not and reconcile this discrepancy.