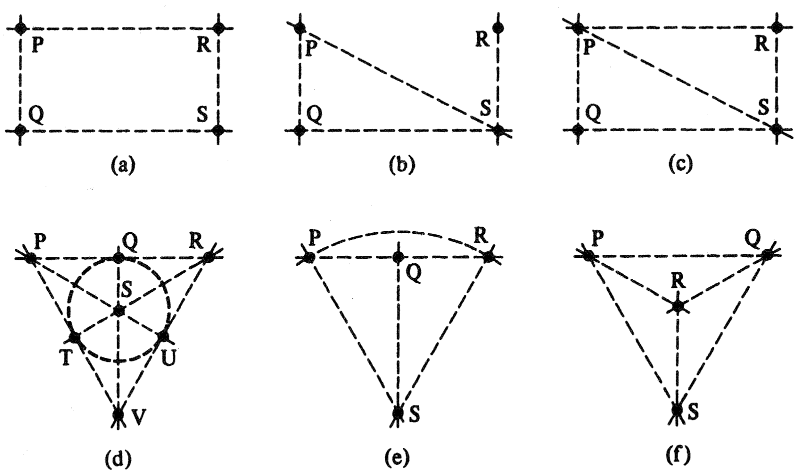
Math 331: Homework 1
Due: Friday, September 5th

1 (2.1.7) Prove that if P and Q are distinct points in $\mathbb{H}$, then they cannot lie simultaneously on both ${}_aL$ and ${}_cL_r$ for some choice of a, c, and r.

2 (2.1.19) Some finite geometries are defined pictorially:



Answer the following questions:

(a) In each example list the set of lines.

(b) Which of these geometries are abstract geometries?

(c) Which of these geometries are incidence geometries?

3 (2.1.20) Let $\{\mathcal{S}, \mathcal{L}\}$ be an abstract geometry and assume that $\mathcal{S}_1 \subseteq \mathcal{S}$. We define an $\mathcal{S}_1$-line to be any subset of $\mathcal{S}_1$ of the form $l \cap \mathcal{S}_1$ where $l \in \mathcal{L}$ and where $l \cap \mathcal{S}_1$ has at least two points. Let $\mathcal{L}_1$ be the collection of all $\mathcal{S}_1$-lines. Prove that $\{\mathcal{S}_1, \mathcal{L}_1\}$ is an abstract geometry. $\{\mathcal{S}_1, \mathcal{L}_1\}$ is called the **geometry induced** from $\{\mathcal{S}, \mathcal{L}\}$.

4 (2.1.21) If $\{\mathcal{S}_1, \mathcal{L}_1\}$ is the geometry induced from an incidence geometry $\{\mathcal{S}, \mathcal{L}\}$, prove that $\{\mathcal{S}_1, \mathcal{L}_1\}$ is an incidence geometry if $\mathcal{S}_1$ has a set of three non-collinear points.

5 (2.1.24) Let $\{\mathcal{S}, \mathcal{L}\}$ be an abstract geometry. If l_1 and l_2 are lines in $\mathcal{L}$, we write $l_1 \sim l_2$ if l_1 is parallel to l_2. Prove or disprove that $\sim$ is an equivalence relation. If $\{\mathcal{S}, \mathcal{L}\}$ is the Cartesian Plane then each equivalence class can be characterized by a real number or infinity. What is this number?

6 (2.2.2) Prove that the function d_H defined by Equations (2-2) and (2-3) on page 28 of your text satisfies axioms (i) and (iii) of the definition of a distance function.

7 (2.2.8) Prove that the function $g : {}_aL \to \mathbb{R}$ given by $g(a, y) = \ln(y)$ is a bijection and that it satisfies the Ruler Equation. Also show that the inverse of g is given by $g^{-1}(t) = (a, e^t)$.

8 (2.2.18) Define the **max distance** (or **supremum distance**), d_S on $\mathbb{R}^2$ by

$$d_S(P, Q) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$$

where $P = (x_1, y_1)$ and $Q = (x_2, y_2)$.

(a) Prove that d_S is a distance function.

(b) Prove that $\{\mathbb{R}^2, \mathcal{L}_E, d_S\}$ is a metric geometry.

9 (2.2.19) In a metric geometry $\{\mathcal{S}, \mathcal{L}, d\}$ if $P \in \mathcal{S}$ and $r > 0$, then the **circle with center *P*** and **radius *r*** is

$$\mathcal{C} = \{Q \in \mathcal{S} : d(P, Q) = r\}.$$

Draw a picture of the circle of radius 1 and center $(0, 0)$ in $\mathbb{R}^2$ for each of the distances d_E, d_T, and d_S.

10 (2.3.2) In the Poincaré Plane find a ruler f with $f(P) = 0$ and $f(Q) > 0$ for the given pair P and Q:

(a) $P = (2, 3)$, $Q = (2, 1)$.

(b) $P = (2, 3)$, $Q = (-1, 6)$.

11 (2.3.5) Prove that a line in a metric geometry has infinitely many points.