Math 331: Homework 2
Due: Friday, September 19

1 (2.3.4) Let P and Q be points in a metric geometry. Show that there is a point M such that $M \in \overleftrightarrow{PQ}$ and $d(P, M) = d(M, Q)$.

2 (3.1.3) Prove that if L_{AB} is a Cartesian line then $f : L_{AB} \to \mathbb{R}$ defined by

$$f(A + t(B - A)) = t\|B - A\|$$

is a ruler for $\{\mathbb{R}^2, \mathcal{L}_E, d_E\}$.

3 (3.1.9) Define a function d_F for points P and Q in $\mathbb{R}^2$ by

$$d_F(P, Q) = \begin{cases} 0 & \text{if } P = Q \\ d_E(P, Q) & \text{if } L_{PQ} \text{ is not vertical} \\ 3d_E(P, Q) & \text{if } L_{PQ} \text{ is vertical} \end{cases}$$

(a) Prove that d_F is a distance function on $\mathbb{R}^2$ and that $\{\mathbb{R}^2, \mathcal{L}_E, d_F\}$ is a metric geometry.

(b) Prove that the triangle inequality is not satisfied for this distance d_F.

4 (3.2.5) Prove that in the Euclidean plane A—B—C if and only if there is a number t with $0 < t < 1$ and $B = A + t(C - A)$.

5 (3.2.6) If A—B—C—D in a metric geometry, prove that $\{A, B, C, D\}$ is a collinear set.

6 (3.3.4) Prove that "congruence" is an equivalence relation on the set of all lines segments in a metric geometry.

7 (3.3.11) Suppose that A and B are distinct points in a metric geometry. $M \in \overleftrightarrow{AB}$ is called a **midpoint** of $\overline{AB}$ if $AM = MB$.

(a) If M is a midpoint of $\overline{AB}$, prove that A—M—B.

(b) If $A = (0, 4)$ and $B = (0, 1)$ are points on the Poincaré Plane, find a midpoint M of $\overline{AB}$. Sketch A, B, and M, on a graph. Does M look like a midpoint?

8 (3.3.12) If A and B are distinct points of a metric geometry, prove that:

(a) The segment $\overline{AB}$ has a midpoint M.

(b) The midpoint M of $\overline{AB}$ is unique.

9 (3.3.16) Prove or disprove that in a metric geometry, $\overrightarrow{AB}$ is the set of all points $C \in \overleftrightarrow{AB}$ such that A is not between C and B.

10 (3.4.1) Prove that $\angle ABC = \angle CBA$ in a metric geometry.