Math 331: Homework 3
Due: Monday, October 13th

1 (4.1.1) If $\mathcal{S}_1$ and $\mathcal{S}_2$ are convex sets of a metric geometry, prove that $\mathcal{S}_1 \cap \mathcal{S}_2$ is convex. Prove or disprove that $\mathcal{S}_1 \cup \mathcal{S}_2$ is convex.

2 (4.1.8) Let ℓ be a line in a metric geometry $\{\mathcal{S}, \mathcal{L}, d\}$ which satisfies PSA. We write $P \sim Q$ if P and Q are on the same side of ℓ. Prove that $\sim$ is an equivalence relation on $\mathcal{S} - \ell$. How many equivalence classes are there and what are they?

3 (4.1.9) Let ℓ be a line in a metric geometry which satisfies PSA. If P and Q are on opposite sides of ℓ and if Q and R are on opposite sides of ℓ, then P and R are on the same side of ℓ.

4 (4.1.19) We define a new distance on $\{\mathbb{R}^2, \mathcal{L}_E\}$ as follows: Let $f : L_0 \to \mathbb{R}$ by

$$f(0, y) = \begin{cases} y & \text{if } y \text{ is not an integer} \\ -y & \text{if } y \text{ is an integer.} \end{cases}$$

(a) Prove that f is a bijection.

For every other line in $\mathbb{R}^2$ choose a Euclidean ruler. By Theorem 2.2.8, this collection bijections determines a distance function d_N which makes $\{\mathbb{R}^2, \mathcal{L}_E, d_N\}$ into a metric geometry.

(b) Prove that $\{(0, y) : 1/2 \leqslant y \leqslant 3/2\}$ is convex in the Euclidean Plane but not in $\{\mathbb{R}^2, \mathcal{L}_E, d_N\}$.

(c) In $\{\mathbb{R}^2, \mathcal{L}_E, d_N\}$ what is the segment from $(0, 1/2)$ to $(0, 3/2)$? Show that this set is convex in $\{\mathbb{R}^2, \mathcal{L}_E, d_N\}$ but is not convex in the Euclidean Plane.

5 (4.3.6) Considering $\{\mathbb{R}^2, \mathcal{L}_E, d_N\}$, prove or disprove that PP is satisfied.

6 (4.3.3) Given $\triangle ABC$ and a point P in a metric geometry which satisfies PSA, prove there is a line through P that contains exactly two points of $\triangle ABC$.

7 (4.3.8) Let $\mathbb{R}^3 = \{(x, y, z) : x, y, z \in \mathbb{R}\}$. If $A, B \in \mathbb{R}^3$ define

- $L_{AB} = \{A + t(B - A) : t \in \mathbb{R}\}$.
- $\mathcal{L} = \{L_{AB} : A, B \in \mathbb{R}^3 \text{ and } A \neq B\}$.

- $d(A, B) = \|A - B\|$.

Prove that $\{\mathbb{R}^3, \mathcal{L}, d\}$ is a metric geometry but that it does not satisfy PSA.

8 (4.4.1) Prove that in a metric geometry, $\mathrm{int}(\overrightarrow{AB})$ and $\mathrm{int}(\overline{AB})$ are convex sets.

9 (4.4.6) In a Pasch geometry, given $\triangle ABC$ and points D, E, F such that B—C—D, A—E—C, and B—E—F, prove that $F \in \mathrm{int}(\angle ACD)$.

10 (4.4.15) In a Pasch geometry, prove that $\mathrm{int}(\angle ABC)$ is convex.

11 (4.5.6) Give a "proper" definition of the interior of a convex quadrilateral. Then prove that the interior of a convex quadrilateral is a convex set. Finally, prove or disprove the statement "A convex quadrilateral in a Pasch geometry is a convex set."