Math 331: Homework 5
Due: Wednesday, November 12th

1 (5.3.6) Prove that in a protractor geometry if $\angle ABC$ and $\angle A'BC'$ form a vertical pair, then $\angle ABC \simeq \angle A'BC'$.

2 (5.3.15) Prove that in a protractor geometry $\angle ABC$ is a right angle if and only if there exists a point D with D—B—C and $\angle ABC \simeq \angle ABD$.

3 (6.1.5) Prove that in a neutral geometry every equilateral triangle is **equiangular**; that is, all its angles are congruent.

4 (6.2.4) Prove that if a protractor geometry satisfies ASA, then it also satisfies SAS and thus is a neutral geometry.

5 (6.3.1) Prove that in a metric geometry, $\overline{AB} < \overline{CD}$ if and only if there is a point $G \in \text{int}(\overline{CD})$ with $\overline{AB} \simeq \overline{CG}$.

6 (6.3.2) Prove that in a protractor geometry, $\angle ABC < \angle DEF$ if and only if there is a point $G \in \text{int}(\angle DEF)$ with $\angle ABC \simeq \angle DEG$.

7 (6.3.6) Prove the General Triangle Inequality for a neutral geometry: If A, B, and C are distinct points in a neutral geometry, then $AC \leqslant AB + BC$. Furthermore, equality holds if and only if A—B—C.

8 (6.4.1) In a neural geometry, let $\triangle ABC$ and $\triangle DEF$ be right triangles with right angles at C and F. Prove that if $\overline{AB} \simeq \overline{DE}$ and $\angle A \simeq \angle D$, then $\triangle ABC \simeq \triangle DEF$.

9 (6.5.3) If $\overline{AB}$ is a chord of a circle in a neutral geometry but is not a diameter, prove that the line through the midpoint of $\overline{AB}$ and the center of the circle is perpendicular to $\overline{AB}$.

10 (6.6.1) Let $\overline{AB}$ and $\overline{DE}$ be two chords of the circle $\mathcal{C} = \mathcal{C}_r(C)$ in a neutral geometry. Prove that if $\overline{AB}$ and $\overline{DE}$ are both perpendicular to a diameter of $\mathcal{C}$ at points P and Q with C—P—Q, then $DQ < AP < r$.