Math 446: Homework 0
Due: Wednesday, January 21st

Section 20 of our text states: Every nonempty subset of \( \mathbb{N} \) has a smallest element. This statement is often known as the **Well-Ordering Principle**.

1 (22α) Use the Well-Ordering Principle to prove that every natural number other than 1 is divisible by some prime.

2 (22β) Construct a natural number larger than 1 which is not divisible by any of the prime numbers in a given list of primes \( p_1, p_2, \ldots, p_k \).

3 (22γ) Prove that the number of primes is infinite.

4 (23) Given \( a, b \in \mathbb{Z} \), prove that \( g = (a, b) \) is the smallest positive integer such that

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g = ua + vb
\]

for some integers \( u \) and \( v \).