# Math 446: Homework 6
# Due: Monday, April 20th

**1 (108$\boldsymbol{\alpha}$)** Prove that the sum, $c + \alpha$, and the product, $c\alpha$, of a rational number $c$ and and algebraic number $\alpha$ are algebraic numbers.

**2 (109$\boldsymbol{\alpha}$)** Let $f$ be an irreducible polynomial over $F$, and let $E$ be an extension field of $F$ in which $f$ has a root $\alpha$. Show that $f$ is a minimal polynomial for $\alpha$ over $F$.

**3 (109$\boldsymbol{\beta}$)** Let $F \subseteq E \subseteq D$ be a tower of fields. Let $\alpha \in D$, and let $g$ be a minimal polynomial for $\alpha$ over $E$ and $f$ a minimal polynomial for $\alpha$ over $F$. Show that $g \mid f$, considering both as polynomials over $E$.

**4 (109$\boldsymbol{\gamma}$)** Find minimal polynomials over $\mathbb{Q}$ and $\mathbb{Q}(\sqrt{2})$ for the numbers $\sqrt{2} + \sqrt{3}$ and $i\sqrt{2}$.

**5 (110)** Let $f$ be a minimal polynomial for $\alpha$ over $\mathbb{Q}$ and $g$ be a minimal polynomial for $\beta$ over $\mathbb{Q}(\alpha)$. Prove that:

$$[\mathbb{Q}(\alpha, \beta) : \mathbb{Q}] = \deg(f) \cdot \deg(g)$$

**6 (110)** Prove that if $[F(\alpha) : F]$ is odd, then $F(\alpha) = F(\alpha^2)$. Hint: What is the set theoretic relationship between $F$, $F(\alpha)$, and $F(\alpha^2)$?

**7 (112$\boldsymbol{\alpha}$)** Prove that the set of all algebraic numbers is a subfield of $\mathbb{C}$. Show that the set of all algebraic numbers is countable. Note that since $\mathbb{C}$ is not countable, this proves the existence of transcendental numbers.

**8 (112$\boldsymbol{\beta}$)** Prove that if $\alpha$ and $\beta$ are transcendental numbers, then $\alpha + \beta$ or $\alpha\beta$ is transcendental.

**9 (121)** Is the regular 9-gon constructible? If so give a construction. If not, give a proof that it is not constructible.