Review of Results from Thursday, 8/12/04

We proved the following important results: a condition under which an infinite series can be differentiated term by term, and a (different) condition under which an infinite series can be integrated term-by-term.

Both of these conditions are sufficient but not necessary.

Theorem 1 (Theorem 5.7) Suppose

1. \( \sum_{k=1}^{\infty} f_k(x) \) converges at \( x = a \)
2. Every \( f_k \) is differentiable on some open interval \( I \) containing \( a \)
3. The series \( \sum_{k=1}^{\infty} f_k' \) converges uniformly on \( I \).

Then \( \sum_{k=1}^{\infty} f_k \) is differentiable at \( a \) and

\[
\frac{d}{dx} \bigg|_{x=a} \left( \sum_{k=1}^{\infty} f_k \right) = \sum_{k=1}^{\infty} f_k'(a).
\]

Corollary 1 (Corollary 5.1) If

1. \( \sum_{k=1}^{\infty} f_k' \) converges uniformly in a bounded interval \( I \)
2. \( \sum_{k=1}^{\infty} f_k(x) \) converges for some \( x \in I \)

then \( \sum_{k=1}^{\infty} f_k \) converges uniformly on \( I \).

Theorem 2 (Theorem 5.8) Suppose \( \sum_{k=1}^{\infty} f_k \) converges uniformly on the interval \( [a, b] \) and each \( f_k \) is integrable over \( [a, b] \). Then

\[
\int_a^b \sum_{k=1}^{\infty} f_k(x) \, dx = \sum_{k=1}^{\infty} \int_a^b f_k(x) \, dx.
\]