Math 650

Summer 2004

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Useful web pages. David Bressoud’s home page: http://www.macalester.edu/~bressoud A Radical Approach to Real Analysis online edition: http://www.macalester.edu/aratra/ Corrections to the second printing of the textbook:
http://www.macalester.edu/~bressoud/books/aratra-correct.html

Mathematica. The textbook uses the computer software Mathematica for a number of exercises. Mathematica is a very powerful symbolic computation package. It is site licensed to OSU students. The license costs $30 and runs from Sept. 25 to Sept. 24 of each year (so if you purchase it today the license expires on Sept. 24, 2004). You can download the software for Macintosh (OSX 10.2 or higher), Windows (98/Me/NT/2000/XP), or Linux (Redhat 7.2 or equivalent or later). It can be downloaded at http://softwaretogo.osu.edu.

OSU also has a site license for the software Maple. It is comparable to Mathematica but has slightly different syntax. The site license for Maple is free, but you have to get the install CD’s from Baker Systems Room 512. See http://oit.osu.edu/site_license. If you know Maple well then you can probably figure out how to convert the Mathematica code in the textbook to Maple.

Problem sets. I’ll give out weekly problem sets.

Tests. There will be two tests and a final exam. See the outline below.

Grades. Grades will be based on problem sets, tests and the final exam. There will be two tests at 100 points each, problem sets worth 100 points, and a final exam worth 150 points, for a total of 450 points. Roughly 85% will correspond to an A, 75% to a B, 65% to a C, 55% to a D, anything below 55% an E.

Office hours. I’ll be in my nominal office for an hour or longer right after class. Other meetings times can easily be arranged.

Accommodations for students with disabilities. Anyone who has need of accommodation based on a disability should contact me as soon as possible.

Outline.

1 Crisis in Mathematics: Fourier’s Series

1.1 Background to the Problem
1.2 Solution and Objections

2 Infinite Summations
2.1 Avoiding Infinite Series

2.2 Newton on $\pi$. Binomial series.

2.3 Logarithms and the Harmonic Series. Historical development of logarithm, Euler’s constant, Nested Interval Principal, series development of exponential.

2.4 Taylor Series. “[Infinite Series] are recognized as the central pillar of calculus.” Series as solutions to differential equations. Taylor’s formula. Convergence, remainder (Lagrange form).

2.5 Emerging Doubts. Divergent series can have different values! Vibrating string problem, Cauchy’s counterexample.

Test 1: Wednesday, July 7.

3 Differentiability and Continuity


3.2 Differentiability Derivative in terms of $\epsilon$ and $\delta$; what can go wrong.

3.3 Cauchy and the Mean Value Theorems.

3.4 Continuity.

3.5 Consequences of Continuity. Darboux property.

4 Convergence of Infinite Series

4.1 The Basic Tests. The basic convergence tests and some interesting motivating examples.

4.2 Series of Functions. Introduction to series of functions. Power series, radius of convergence. lim sup, lim inf.

4.3 Hypergeometric Series. Gauss’s test for convergence.

4.4 The Finer Tests. Cauchy’s Condensation test, integral test.


Test 2: Friday, August 6.

5 Understanding Infinite Series

5.1 Groupings and Rearrangements.

5.2 Cauchy and Continuity. Uniform convergence.

5.3 Differentiation and Integration. What goes wrong when you try to differentiate a series term-by-term, or integrate it term by term. When is it permissible to differentiate term-by-term? To integrate term-by-term?
5.4 Verifying Uniform Convergence. Weierstrass $M$-test. Application to power series. Implications for convergence of power series at end points of interval of convergence. Dirichlet’s test for uniform convergence.

6 Return to Fourier Series

6.1 Dirichlet’s Theorem.

Final Exam: Thursday, August 26, 3:30–5:18 pm.