Sample Test 2, Math 551, Tanveer
Instructions: Closed book and notes. Show work

1a. In terms of a double integral with appropriate limits determine an expression for the surface area of the part of the surface \( z = x^2 + y^2 \) that is inside the cylinder \( x^2 + y^2 = 4 \).

1b. With appropriate changes of variable determine the area integral \( \int \int_R \sin(2x) dA \), where \( R \) is the region bounded by \( x - y = 0 \), \( x - y = 1 \), \( x + y = 0 \), \( x + y = 2 \).
2a. Determine
\[ \int_C xdx + xdy + ydz \]
where \( C \) is a straight line from \((0, 2, 3)\) to \((-1,3,0)\).

2b. Determine if the answer in (1b) will generally depend on the path connecting the two given points.
3a. Determine if $\mathbf{F} = (ye^{xy} - 2x)\mathbf{i} + (xe^{xy} + 2y)\mathbf{j}$ is conservative. If so, determine the scalar potential $f$ so that $\mathbf{F} = \nabla f$.

3b. Determine the upwards flux of the vector field $\mathbf{F} = z\mathbf{k}$ across the upper hemisphere $z = \sqrt{1 - x^2 - y^2}$. 
4a. Determine the outwards flux of the vector field
\[ \mathbf{F} = z \sin y \mathbf{i} + e^x \mathbf{j} + xy \mathbf{k} \]
through the surface \( \partial S \) of the solid bounded by \( z = 0 \) and \( z = 4 - \sqrt{x^2 + y^2} \).

4b. Calculate \( \oint_C \mathbf{F} \cdot d\mathbf{x} \) where
\[ \mathbf{F} = 2z \mathbf{i} + 6x \mathbf{j} - 3y \mathbf{k} \]
and \( C \) is the counter-clockwise closed path \( x^2 + y^2 = 1, \ z = 0 \).
5a. For the vector field \( \mathbf{F} = 3xy \mathbf{i} + y^2 \mathbf{j} - x^2 y^4 \mathbf{k} \), calculate \( \oint_C \mathbf{F} \cdot d\mathbf{x} \), where \( C \) is the counter-clockwise closed path \( x^2 + y^2 = 1, z = 0 \).

5b. For the vector field \( \mathbf{F} \) in (4a), determine
\[
\int \int_M (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dA
\]
where \( M \) is the surface of the part of the ellipsoid \( x^2 + y^2 + 2z^2 = 1 \) with \( z \geq 0 \) and \( \mathbf{n} \) is taken to have a positive \( \mathbf{k} \) component.