Solution to Set 2, due Friday April 9

(1) (Section 1.4, Problem 1b) Plot the point \((1, 0, -4)\) in cylindrical coordinate.

**Solution:** Note that \(r = 1, \theta = 0\), implies \(x = 1 \cos 0 = 1, y = 1 \sin 0 = 0\). Since \(w = -4, z = -4\). So, the point in question has cartesian (rectangular) coordinates \((1, 0, -4)\) and the plot looks like in the figure.

![Diagram of a cylindrical coordinate system with point (1,0,-4) marked](image)

**Figure 1.** Point \((1,0,-4)\) in cylindrical coordinate

(2) (Section 1.4, Problem 2c) Plot the point with spherical coordinates \(\left(\frac{1}{2}, \frac{3}{2}\pi, \frac{3}{4}\pi\right)\).

**Solution:** Noting that \(\rho = \frac{1}{2}, \theta = \frac{3}{2}\pi\) and \(\phi = \frac{3}{4}\pi\), it follows that

\[
x = \frac{1}{2} \sin \left(\frac{3\pi}{4}\right) \cos \left(\frac{3\pi}{2}\right) = 0
\]

\[
y = \frac{1}{2} \sin \left(\frac{3\pi}{4}\right) \sin \left(\frac{3\pi}{2}\right) = -\frac{1}{2\sqrt{2}}
\]

\[
z = \frac{1}{2} \cos \left(\frac{3\pi}{4}\right) = -\frac{1}{2\sqrt{2}}
\]

So, the cartesian coordinate of the point in question is \(\left(0, -\frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right) \approx (0, -0.353, -0.353)\).

(3) (Section 1.4, Problem 4c) Find rectangular coordinates of the point in exercise 2c.
Figure 2. Point with spherical coordinate $\left(\frac{1}{2}, \frac{3}{2}\pi, \frac{3}{4}\pi\right)$.

Solution: In the process of plotting the point, we found in (2c.) that this point corresponds to rectangular coordinate $(0, -\frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{2}) \approx (0, -0.353, -0.353)$.

(4) (Section 1.4, Problem 6) Write the following equations in cylindrical coordinates and sketch the graph:

Solution to a.: $x^2 + y^2 + (z - 1)^2 = 1$ sphere of radius 1 centered at $(0, 0, 1)$. To get equation in cylindrical coordinate, we note $(z - 1)^2 = 1 - x^2 - y^2 = 1 - r^2$; so $z = 1 \pm \sqrt{1 - r^2}$ is the equation of the sphere.

Figure 3. Sketch of sphere $x^2 + y^2 + (z - 1)^2 = 0$. 
Solution to b.: $z = 2(x^2 + y^2)$ implies $z = 2r^2$. This corresponds to an elliptic paraboloid opening upwards with lowest point at $(0,0,0)$. Plot easiest by drawing curve on the $z-y$ plane and rotate around the $x$-axis to obtain the surface of revolution.

![Figure 4. Sketch of sphere $z = 2(x^2 + y^2) = 2r^2$.](image)

Solution to c.: $x = 1$ is a plane parallel to the $y-z$ plane that in cylindrical coordinates is described by $r \cos \theta = 1$, or $r = \sec \theta$.

![Figure 5. Sketch of plane $x = 1$ or $r = \sec \theta$](image)

Solution to d.: $z = -3$ is a plane that is parallel to the $x-y$ plane but shifted down by 3. In the cylindrical coordinate system, this is given by $w = -3$. 

Solution to e. $x^2 + y^2 + z^2 = 100 = 10^2$ is clearly a sphere of radius 10 about the origin. In cylindrical coordinates, we get $r^2 + w^2 = 100$.

(5) (Problem 8, Section 1.4) Convert the following spherical coordinate equations to Cartesian coordinates and sketch the graph.

Solution to a: $\rho = 18$ is clearly a sphere of radius 18 about the origin, and $\rho = \sqrt{x^2 + y^2 + z^2} = 18$. Therefore, $x^2 + y^2 + z^2 = 18^2 = 324$ is the equation of sphere in cartesian coordinates.
Figure 8. Sketch of $\rho = 18$ or $x^2 + y^2 + z^2 = 324$.

Solution to b. $\phi = \frac{\pi}{6}$ is a cone obtained by rotating the half-line from the origin in the $x-z$ plane that makes an angle $\frac{\pi}{6}$ with respect to $z$-axis and rotating around $z$ axis to obtain the cone as a surface of revolution. In the cartesian coordinate system, $\frac{\sqrt{3}}{2} = \cos \frac{\pi}{6} = \frac{z}{\sqrt{x^2+y^2+z^2}}$, or $x^2 + y^2 + z^2 = \frac{4}{3}z^2$ or $z^2 = 3(x^2 + y^2)$. But since we are concerned only with $z > 0$ since $\cos \phi = \cos \frac{\pi}{6} > 0$, we have $z = \sqrt{3(x^2 + y^2)}$.

Figure 9. Sketch of cone $\phi = \frac{\pi}{6}$.

Solution to c. $\theta = \frac{\pi}{4}$ corresponds to the half-plane one obtains by rotating the half-plane in the $x-z$ plane (restricted to $x > 0$) by $45^\circ$ about the $z$-axis. Also, this can be seen by converting to cartesian coordinates. Since $\tan \theta = 1 = \frac{y}{x}$, get $y = x$ plane with restriction $x > 0$. 
Solution to d. $\rho = 4 \sec \phi$ implies $\rho \cos \phi = 4$ or $z = 4$, which is a plane parallel to the $x - y$ plane shifted up by 4.

Solution to e. $\rho = 5 \cos \phi$ or on multiplying by $\rho$ both sides of the equation, $\rho^2 = 5 \rho \cos \phi$, or $x^2 + y^2 + z^2 = 5z$ or on completing the square $x^2 + y^2 + (z^2 - 5z + \frac{25}{4}) = \frac{25}{4}$, or

$$x^2 + y^2 + \left(z - \frac{5}{2}\right)^2 = \frac{25}{4},$$

sphere of radius $\frac{5}{2}$ about $(0, 0, \frac{5}{2})$: 

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**Figure 10.** Sketch of half-plane $\theta = \frac{\pi}{4}$

**Figure 11.** Sketch of plane $\rho = 4 \sec \phi$ or $z = 4$ plane
(6) (Section 1.5, Problem 1a) Calculate $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}$ when $\mathbf{a} = 5\hat{i} - \hat{j}$, $\mathbf{b} = 7\hat{i} - 3\hat{j} + 2\hat{k}$, $\mathbf{c} = -4\hat{i} + 9\hat{j} - 8\hat{k}$ and $\mathbf{d} = -3\hat{i} - 11\hat{j} + 7\hat{k}$.

Solution: Adding each component, we have

$$\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = 5\hat{i} - 6\hat{j} + \hat{k}$$

(7) (Section 1.5, Problem 1b) For $\mathbf{a} = 5\hat{i} - \hat{j}$, $\mathbf{b} = 7\hat{i} - 3\hat{j} + 2\hat{k}$,

$$\|\mathbf{a}\| - \|\mathbf{b}\| = \sqrt{5^2 + 1^2} = \sqrt{26} - \sqrt{7^2 + 3^2 + 2^2} = \sqrt{26} - \sqrt{50}$$

(8) (Section 1.5, Problem 3)

a. Find a vector opposite to $\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}$. Answer: $-\hat{i} - 2\hat{j} - 3\hat{k}$.

b. Find a vector with tail at the point (-8,2,7) and head at the point (10, 12, 17). Answer: Subtracting the tail coordinates from the head, we have the vector $18\hat{i} + 10\hat{j} + 10\hat{k}$, or equivalently $(18, 10, 10)$.

c. A unit vector parallel to $\mathbf{a} = 4\hat{i} + 2\hat{j} - \hat{k}$. Answer: $\frac{1}{\sqrt{4^2 + 2^2 + (-1)^2}}(4, 2, -1) = \left(\frac{4}{\sqrt{21}}, \frac{2}{\sqrt{21}}, -\frac{1}{\sqrt{21}}\right)$.

d. A vector of magnitude 13 parallel to $5\hat{i} + 2\hat{k}$. Answer: $\frac{13}{\sqrt{5^2 + 2^2}}(5, 0, 2) = \left(\frac{65}{\sqrt{29}}, 0, \frac{26}{\sqrt{29}}\right)$.

e. A unit vector that in standard position is at an angle 60° with both x and y axes. Answer: Let the vector be $(a, b, c)$ with $a^2 + b^2 + c^2 = 1$. We have $a = \cos 60° = \frac{1}{2}$, $b = \cos 60° = \frac{1}{2}$. 

Figure 12. Sketch of sphere $\rho = 5 \cos \phi$ or $x^2 + y^2 + (z - 5/2)^2 = 25/4$. 

\[
\begin{align*}
x^2 + y^2 &= \frac{2}{25/4} \\
(z - 5/2) &= 25/4
\end{align*}
\]
So, \( c = \sqrt{1 - \frac{1}{4} - \frac{1}{4}} = \frac{1}{\sqrt{2}} \). So one choice of unit vector is \( \left( \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \). We could have replaced \( 1/\sqrt{2} \) in the third component by \( -1/\sqrt{2} \). Both answers are acceptable.

(9) (Problem 8, Section 1.5) In the following equations, solve for the unknowns

a. \( t \hat{i} + 3 \hat{j} + \hat{k} = (4t + 3)\hat{i} + 3\hat{j} + \hat{k} \). Answer: Bringing everything on one side \((t - 4t - 3)\hat{i} = 0\), so \( t = -1 \).

b. \((s, -2t) = (1 - t, s + 2)\). Answer: Bringing everything on one side, \((s - 1 + t, -2t - s - 2) = (0, 0)\). So, \( s = 1 - t \) and using this in the second equation \( 2t = -s - 2 = t - 3 \), obtain \( t = -3, s = 4 \).

c. \((s, t, u) = (0, 5, 5)\). Answer: Bringing everything on one side, \((s, t - 5, u - 5) = (0, 0, 0)\). So \( s = 0, t = 5 \) and \( u = 5 \).

d. \( s(\hat{i} + \hat{j} + \hat{k}) = 4t\hat{i} + 3u\hat{j} + 8\hat{k} \). Answer: Multiplying each component by \( s \) and bringing everything on one side, we have \((s - 4t, s - 3u, s - 8) = (0, 0, 0)\). Therefore \( s = 8, s = 4t, i.e. t = 2 \) and \( 3u = s = 8, i.e. u = \frac{8}{3} \).

(10) (Problem 20, Section 1.5). A median of a triangle is a line segment joining a vertex to the midpoint of the opposite side. Using vectors, show that the point in which two medians intersect cuts them both into two segments such that the lengths of the subsegments are in the ratio 2:1 (See Figure below).

**Solution:** Define \( \mathbf{a} = \mathbf{AB} \) and \( \mathbf{b} = \mathbf{AC} \). Then, for some scalar \( t \) to be determined, we have

\[ \mathbf{AF} = t\mathbf{AD} = t \left[ \mathbf{AB} + \frac{1}{2} \mathbf{BC} \right] = t \left[ \mathbf{a} + \frac{1}{2} (\mathbf{b} - \mathbf{a}) \right] = \frac{t}{2} \mathbf{a} + \frac{t}{2} \mathbf{b} \]

Again for some scalar \( s \), \( \mathbf{AF} = \mathbf{AE} + \mathbf{EF} = \mathbf{AE} + s \mathbf{EB} \) or

\[ \mathbf{AF} = \frac{1}{2} \mathbf{b} + s \left( \mathbf{a} - \frac{1}{2} \mathbf{b} \right) = sa + \frac{(1 - s)}{2} \mathbf{b} \]

Equating the two expressions above for \( \mathbf{AF} \),

\[ \frac{t}{2} \mathbf{a} + \frac{t}{2} \mathbf{b} = sa + \frac{(1 - s)}{2} \mathbf{b} \]

Bringing over everything to one side

\[ \left( \frac{t}{2} - s \right) \mathbf{a} + \left( \frac{t}{2} - \frac{(1 - s)}{2} \right) \mathbf{b} = 0 \]

Since vectors \( \mathbf{a} \) and \( \mathbf{b} \) are linearly independent, it follows that \( t/2 = s \) and \( t/2 - (1 - s)/2 = 0 \). So, \( t = 2s \). Substituting into second expression \( s - (1 - s)/2 = 0 \), i.e. \( \frac{3}{2} s = \frac{1}{2} \). So, \( s = \frac{1}{3} \).
and $t = \frac{2}{3}$. Since we took $\mathbf{AF} = t\mathbf{AD} = \frac{2}{3}\mathbf{AD}$ it follows that the ratio of lengths $\|\mathbf{AF}\|$ and $\|\mathbf{FD}\|$ is $2 : 1$. Again we took $\mathbf{EF} = s\mathbf{EB} = \frac{1}{3}\mathbf{EB}$, it follows ratio of lengths $\|\mathbf{FB}\|$ and $\|\mathbf{EF}\|$ is also $2 : 1$.

![Figure 13. Medians $AD$ and $BE$ of a triangle $ABC$ intersecting at $F$](image)

(11) (Section 1.6, Problem 1c.) Calculate dot product of $\mathbf{a} = 6\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} - \mathbf{j} + \mathbf{k}$. Answer: $\mathbf{a} \cdot \mathbf{b} = (6)(-1) + (4)(-1) + (-2)(1) = -12$.

(12) (Section 1.6, Problem 3). For each expression, determine whether or not it makes sense. Explain in either case.

a. $\|\mathbf{a}\|\mathbf{b} - \|\mathbf{b}\|\mathbf{a}$. Yes it makes sense since $\|\mathbf{a}\|$ and $-\|\mathbf{b}\|$ are merely scalars and scalar multiplication of vectors is well-defined as is sum of two vectors.

b. $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$. Note dot product involves two vectors. While $(\mathbf{b} \cdot \mathbf{c})$ makes sense, this result in a scalar which cannot have a dot product with a vector $\mathbf{a}$. So, the entire expression is nonsense.

c. $(\mathbf{a} - \mathbf{b}) \cdot \mathbf{c}/2$ makes sense. Note $\mathbf{a} - \mathbf{b}$ is a vector as is $\mathbf{c}/2$ (scalar multiple of $\frac{1}{2}$ multiplying $\mathbf{c}$). So, at the end dot product makes sense.

d. $\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{c}}$. This makes no sense since division by a vector $\mathbf{c}$ is not defined.

e. $\|\mathbf{a}\|^2\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})^2\mathbf{a}$. This makes perfect sense since $\|\mathbf{a}\|^2$ and $\mathbf{a} \cdot \mathbf{b}$ are each scalar and scalar multiplication of vectors and addition of vectors are well-defined.

(13) (Section 1.6, Problem 4). In the following, calculate $\text{comp}_\mathbf{b}\mathbf{a}$ and $\text{proj}_\mathbf{b}\mathbf{a}$. 

(a.) \( \mathbf{a} = (4, 6, -5), \mathbf{b} = (-1, -1, -1). \)

\[
\text{comp}_b \mathbf{a} = \frac{(4, 6, -5) \cdot (-1, -1, -1)}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{-4 - 6 + 5}{\sqrt{3}} = -\frac{5}{\sqrt{3}}
\]

\[
\text{proj}_b \mathbf{a} = -\frac{5}{\sqrt{3}} \cdot \frac{(-1, -1, -1)}{\sqrt{3}} = \left( \frac{5}{3}, \frac{5}{3}, \frac{5}{3} \right)
\]

\[ b. \ \mathbf{a} = \left( \frac{1}{\sqrt{2}}, -\sqrt{2}, \frac{1}{\sqrt{2}} \right), \mathbf{b} = (1, 1, 1). \]

\[
\text{comp}_b \mathbf{a} = \left( \frac{1}{\sqrt{2}}, -\sqrt{2}, \frac{1}{\sqrt{2}} \right) \cdot \frac{(1, 1, 1)}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}} \left( \frac{1}{\sqrt{2}} - \sqrt{2} + \frac{1}{\sqrt{2}} \right) = 0
\]

and so

\[ \text{proj}_b \mathbf{a} = 0 \]

\[ c. \ \mathbf{a} = (4, 8, 16), \mathbf{b} = (3, 9, 27). \]

\[
\text{comp}_b \mathbf{a} = \frac{(4)(3) + (8)(9) + (16)(27)}{\sqrt{3^2 + 9^2 + 27^2}} = \frac{516}{\sqrt{819}}
\]

\[
\text{proj}_b \mathbf{a} = \frac{516}{819} (3, 9, 27) = \left( \frac{516}{273}, \frac{516}{91}, \frac{1548}{91} \right)
\]

(14) (Section 1.6, Problem 6). Find \( t \) so that \( \mathbf{a} = (t, 1, 7) \) is perpendicular (orthogonal) to \( \mathbf{b} = (2, -2, 1) \). Answer: Require \( 0 = \mathbf{a} \cdot \mathbf{b} = 2t - 2 + 7 = 0 \). So \( t = -\frac{5}{2} \).

(15) (Section 1.6, Problem 22) Find the work done by gravity as a 1 kg body is pushed 5m up an inclined plane that is sloped up \( 36^\circ \) from horizontal (see Fig. below)

**Figure 14.** Schematic of 1 kg body pushed up an inclined plane 5 meters
Solution: Note Gravity force $\mathbf{F}_g = mg$ and displacement $\mathbf{d}$ makes an angle of $126^\circ$ to each other. Therefore,

$$\text{Work} = (1)(9.8)(5)\cos(126^\circ) \text{Joules} = -28.8 \text{Joules}$$

(16) (Section 1.7, Problem 1d.) $\mathbf{a} = \frac{1}{2}\hat{i} + \frac{1}{3}\hat{j} - \frac{1}{5}\hat{k}$ and $\mathbf{b} = 15\hat{i} + 10\hat{j} - 6\hat{k}$. Calculate $\mathbf{a} \times \mathbf{b}$.

Solution:

$$\det \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & -\frac{1}{5} \\ 10 & 15 & -6 \end{pmatrix} = \hat{i}(-2 + 2) - \hat{j}(-3 - (-3)) + \hat{k}(5 - 5) = 0$$

(17) (Section 1.7, Problems 2a, 2b)

a. Show by an example that, in general, $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$. Answer:

We can check from cross product formula that $\hat{i} \times \hat{j} = \hat{k}$, but $\hat{j} \times \hat{i} = -\hat{k}$. So, we have examples showing that in general $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$.

b. Show by example that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \neq (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$.

Note that if we take $\mathbf{a} = \hat{i}$, $\mathbf{b} = \hat{i}$ and $\mathbf{c} = \hat{j}$, then since $\hat{i} \times \hat{j} = \hat{k}$ we obtain $\hat{i} \times (\hat{i} \times \hat{j}) = \hat{i} \times \hat{k} = -\hat{j}$. However,

$$(\hat{i} \times \hat{i}) \times \hat{j} = 0 \times \hat{j} = 0.$$ Therefore, in general

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \neq (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$$

(18) (Section 1.7, Problem 5) Prove that $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$.

Proof. We take $\mathbf{a} = (a_1, a_2, a_3)$, $\mathbf{b} = (b_1, b_2, b_3)$ and $\mathbf{c} = (c_1, c_2, c_3)$. We note that $\mathbf{b} + \mathbf{c} = (b_1 + c_1, b_2 + c_2, b_3 + c_3)$.

So, we obtain

$$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{pmatrix} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix} + \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$$

(19) (Section 1.7, Problem 22.) Find the area of the parallelogram in $\mathbb{R}^3$ spanned by $\mathbf{a} = 4\hat{i} + 2\hat{j} - \hat{k}$ and $\mathbf{b} = -\hat{i} + 10\hat{j} + 7\hat{k}$.

Solution. The area of the parallelogram is simply $\| \mathbf{a} \times \mathbf{b} \|$. We calculate the cross product first:

$$\mathbf{a} \times \mathbf{b} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & -1 \\ -1 & 10 & 7 \end{pmatrix} = \hat{i}(14 + 10) - \hat{j}(28 - 1) + \hat{k}(40 + 2) = 24\hat{i} - 27\hat{j} + 42\hat{k}$$

So area of the parallelogram is $\sqrt{24^2 + 27^2 + 42^2}$. 

(20) (Section 1.8, Problem 1a,c)

a. Equation of the plane through (0,2,5) perpendicular to $6\hat{i} + \hat{j} - \hat{k}$:

$$6(x - 0) + 1(y - 2) - 1(z - 5) = 0,$$

or $6x + y - z = -3$

c. Equation of the plane through the origin and perpendicular to the line segment from $(3,8,7)$ and $(4, 1, -9)$. Since normal to the plane in this case is $(4-3, 1-8, -9-7) = (1, -7,-16)$, it follows that the equation of the normal plane is

$$1(x - 0) - 7(y - 0) - 16(z - 0) = 0$$

or $x - 7y - 16z = 0$

(21) (Section 1.8, Problem 2 a, c)

a. Find a parametrization of the line through $(0, 5, -1)$ parallel to $-\hat{i} + 4\hat{j} + \hat{k}$. We have the solution $\mathbf{x} = (0, 5, -1) + t(-1, 4, 1)$. So, reading of components of $\mathbf{x} = (x, y, z)$, we have from above

$$x = -t, \quad y = 5 + 4t, \quad z = -1 + t$$

c. Find a parametrization of the line through $(-8, 1, 4)$ and perpendicular to the plane with equation $6y + 6z = 11$. We know normal to the plane is $(0, 6, 6) = \mathbf{m}$. So, parametric equation of the straight line desired is

$$\mathbf{x} = (x, y, z) = (-8, 1, 4) + t(0, 6, 6)$$

So, $x = -8, \ y = 1 + 6t, \ z = 4 + 6t$

(22) (Section 1.8, Problem 8) Find an equation for the plane through the origin that is perpendicular to the planes $5x - y + z - 2 = 0$ and $2x + 2y - 13z = 2$.

**Solution:** Note that if two planes are perpendicular to one another, then their corresponding normals are perpendicular to each other. Therefore, from the statement, it follows that the normal $\mathbf{N}$ to the plane desired is actually normal to both $(5, -1, 1)$ and $(2, 2, -13)$. We take the cross product to find such $\mathbf{N}$:

$$\mathbf{N} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 1 \\ 2 & 2 & -13 \end{pmatrix} = \hat{i}(13-2)-\hat{j}(-65-2)+\hat{k}(10+2) = 11\hat{i}+67\hat{j}+12\hat{k}$$

Therefore, equation of the desired plane is $11x + 67y + 12z = 0$.

(23) (Section 1.8, Problem 13). Find the parametric equation for the line in which the two planes $x+y+z-2 = 0$ and $x+2y+3z+4 = 0$ intersect.

**Solution:** The vector $\mathbf{m}$ to which the desired straight line is parallel is clearly along the intersection of the two planes. Hence
\( \mathbf{m} \) must be perpendicular to each of the two normals of the planes, and hence to (1,1,1) and (1, 2, 3). Therefore,

\[
\mathbf{m} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix} = \hat{i}(3 - 2) - \hat{j}(3 - 1) + \hat{k}(2 - 1) = \hat{i} - 2\hat{j} + \hat{k}
\]

Now, to find a convenient point \( \mathbf{x}_0 \) on the intersection line, we ask for instance where both planes intersect \( z = 0 \). This would mean

\[
x + y - 2 = 0, \text{ and } x + 2y + 4 = 0
\]

Eliminating \( x \), we obtain \( y + 6 = 0 \), or \( y = -6 \). So, \( x = 2 - y = 8 \).

So we have point on the intersection plane \( \mathbf{x}_0 = (8, -6, 0) \). So, equation of straight line on the intersection of two planes is given by

\[
\mathbf{x} = (x, y, z) = \mathbf{x}_0 + t\mathbf{m} = (8, -6, 0) + t(1, -2, 1)
\]

So, \( x = 8 + t \), \( y = -6 - 2t \), \( z = t \)