Homework Set 6: Math 6451, Due Friday, April 7th, 2017

1. Prove Lemma 6, week 10 notes.

2. Using results we have obtained in class, prove that the solution to Poisson equation

$$\Delta u = f \quad \text{for } \mathbf{x} \in \Omega \subset \mathbb{R}^n, \text{ with } u(\mathbf{x}) = g \quad \text{on } \partial\Omega \tag{1}$$

exists when $g \in C^0(\partial\Omega)$ and $f \in C^0(\overline{\Omega})$, where Ω is bounded with $\partial\Omega$ regular enough to apply Lemma 14 in week 10 notes.

3. Consider

$$u_t + u^2 u_x = 0 \ , x \in \mathbb{R} \ , t > 0 \ , \quad \text{with } u(x,0) = F(x)$$

Find a candidate for weak solution beyond the time when classical solution does not exist and determine conditions on the motion of the shock that guarantees it is a weak solution.