Homework Set 7: Math 6451, Due April 19, 2016

1. Use energy method to prove uniqueness of solution to the initial value problem

$$u_t = \Delta u + u^2 \ \ , \ u(x,0) = \phi(x) \ \ x \in \Omega\mathbb{R}^n \tag{1}$$

   State any assumptions you need to make.

2. Verify the uniform bounds claimed for the L^2 norm of $\partial_x^4 u_n$, where u_n is the Galerkin approximation introduced in Week 12 notes.

3. Using ideas in the proof of viscous Burger's equation, prove there exists solution $u(x,t)$ satisfying the following equation and initial conditions for $x \in \mathbb{R}$, $t \in (0,T]$ for some small enough T:

$$u_t - u_{xx} = u^2 \quad , \quad u(x,0) = F(x)$$