

Homework Set 2: Math 8610, Due: Friday, Sept. 21

1. Show that in 2-D steady flow, a first integral of the ODE $\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x})$ is given by $\psi(x_1, x_2) = C$, implying that level sets of stream functions coincides with particle trajectories (This is not necessarily true for time-dependent flow-fields).
2. Show that in 2-D incompressible flow (not necessarily steady or potential), the flux of fluid flow $\int_C \mathbf{u} \cdot \mathbf{n} ds$ through an arbitrary smooth curve C joining two points is the difference of stream function ψ at the two points.
3. First exercise on Poiseuille flow on page 3, week 4 notes.
4. Second exercise on Poiseuille flow on page 2, week 4 notes.
5. Give details of the argument showing (2.33), week 4 notes and the fact that $\mathcal{N}$ is a contraction map when (2.34) is satisfied.
6. Give details of the argument showing that if (3.51) in week 4 notes holds then $\mathcal{N}$ defined in (2.23) is contractive in the ball B_E in the Banach space $\mathcal{S}_E$.