Homework Set 4: Math 8610, Due: October 27th

1. Show directly from the solution (1.34), week 7 notes, of Stokes flow past a moving sphere through integration of stress on the boundary that the force on the sphere is exactly $-6\pi a\mathbf{U}_0$. **Note:** You might want to use the fact that the i-th force component $F_i = -pn_i + 2\mu S_{ij}n_j$ and on a sphere $n_i = \frac{x_i}{|\mathbf{x}|}$ and pressure is given by (1.24) with $C_1 = -|\mathbf{A}|/(8\pi)$. Use of symmetry will reduce the calculation only to F_3, assuming x_3-axis is directed along $\mathbf{U}_0$.

2. Verify that expressions (1.37)-(1.38) of week 7 notes on the velocity due to a Stokeslet in the presence of a wall. **Note:** We already know from construction that (1.37) is a solution to Stokes flow in the upper-half plane except at $\mathbf{x} = \mathbf{y}$, where we a Stokeslet singularity. You only need to verify that it satisfies no-slip BC $\mathbf{u} = 0$ on $x_3 = 0$.

3. Suppose $\mathbf{b} = 0$ in sound equations (5.42)-(5.43) of week 8 notes and we have a potential flow $\mathbf{u} = \nabla\phi$. Derive wave equation for perturbed pressure p_1. Return to (5.36) and (5.37) for $\mathbf{b} = 0$ and determine conditions so that the the decomposition (5.41) leads to wave equation for perturbed pressure p_1. Be precise.