Homework 1

Exercise 2(b). Let $P, Q$ and $R$ be sentences. Then $P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$.

Proof. Let $P, Q$ and $R$ be sentences. Assume $P \land (Q \lor R)$ is true. Then both $P$ and $Q \lor R$ are true. Since $Q \lor R$ is true, at least one of $Q$ and $R$ is true. Therefore, at least one of $P \land Q$ and $P \land R$ is true. Therefore $(P \land Q) \lor (P \land R)$ is true.

Conversely, assume $(P \land Q) \lor (P \land R)$ is true. Then at least one of $P \land Q$ and $P \land R$ is true.

Case 1. Assume $P \land Q$ is true. Therefore both $P$ and $Q$ are true. Since $Q$ is true, $Q \lor R$ is true. Hence, $P \land (Q \lor R)$ is true.

Case 2. Assume $P \land R$ is true. Therefore both $P$ and $R$ are true. Since $R$ is true, $Q \lor R$ is true. Hence, $P \land (Q \lor R)$ is true.

In either case, we have $P \land (Q \lor R)$ is true. □

Exercise 3. $P \land (Q \lor R) \not\equiv (P \land Q) \lor R$.

Proof. Let $P$ be false, $Q$ be true and $R$ be true. Since $P$ is false, $P \land (Q \lor R)$ is false. Since $R$ is true, $(P \land Q) \lor R$ is true. Therefore, $P \land (Q \lor R) \not\equiv (P \land Q) \lor R$. □