Homework 2

Exercise 1(h). \((\forall x \geq 7)(x^2 - 4x + 3 > 0)\) is true.

Proof. Let \(x \geq 7\). Then \(x - 3 \geq 7 - 3 = 4 > 0\) and \(x - 1 \geq 7 - 1 = 6 > 0\). Therefore, 
\[x^2 - 4x + 3 = (x - 3)(x - 1) > 0.\]

Exercise 1(k). \((\exists x \geq 0)(\sqrt{x + 3} = \sqrt{x} + \sqrt{3})\) is true.

Proof. Let \(x = 0\). Then \(x = 0 \geq 0\) and \(\sqrt{x + 3} = \sqrt{0 + 3} = \sqrt{3} = \sqrt{0} + \sqrt{3} = \sqrt{x} + \sqrt{3}\). \(\square\)

Exercise 5(a). \((\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})[\text{if } x < y, \text{ then } (\exists a \in \mathbb{N})(x + a = y)]\) is true.

Proof. Let \(x \in \mathbb{Z}\) and \(y \in \mathbb{Z}\). Suppose \(x < y\). Let \(a = y - x\). Then \(x + a = x + (y - x) = y\).
Since \(x, y \in \mathbb{Z}\) and \(y - x > 0\), \(a = y - x \in \mathbb{N}\). \(\square\)

Exercise 5(b). \((\forall x \in \mathbb{R})(\forall y \in \mathbb{R})[\text{if } x < y, \text{ then } (\exists a \in \mathbb{N})(x + a = y)]\) is false.

Proof. Let \(x = 0\) and \(y = \frac{1}{2}\). Then \(x = 0 < \frac{1}{2} = y\). Let \(a \in \mathbb{N}\). Then \(a \geq 1 > \frac{1}{2} = y - x\).
Hence \(x + a > y\); so \(x + a \neq y\). \(\square\)