Homework 4

Exercise 1(a). If $x$ is even and $y$ is even, then $x + y$ is even.

Proof. Let $x$ and $y$ be even. Then there exist $m, n \in \mathbb{Z}$ such that $x = 2m$ and $y = 2n$. Therefore, $x + y = 2m + 2n = 2(m + n)$. Since $m + n \in \mathbb{Z}$, $x + y$ is even. \hfill \Box

Exercise 2(a). If $x$ is odd and $y$ is odd, then $xy$ is odd.

Proof. Let $x$ and $y$ be odd. Then there exist $m, n \in \mathbb{Z}$ such that $x = 2m + 1$ and $y = 2n + 1$. Therefore, $xy = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1$. Since $2mn + m + n \in \mathbb{Z}$, $xy$ is odd. \hfill \Box