Abstract

We derive key results from dimension theory in dynamical systems and thermodynamic formalism at a level of generality suitable for the study of systems which are beyond the scope of the standard uniformly hyperbolic theory. Let (X, d) be a compact metric space, $f : X \mapsto X$ be a continuous map and $\varphi : X \mapsto \mathbb{R}$ be a continuous function.

The subject of chapters 4 and 5 is the multifractal analysis of Birkhoff averages for φ when topological pressure (in the sense of Pesin and Pitskel) is the dimension characteristic and f has the specification property. In chapter 4, we consider the set of points for which the Birkhoff average of φ does not exist (which we call the irregular set for φ) and show that this set is either empty or has full topological pressure. We formulate various equivalent natural conditions on φ that completely describe when the latter situation holds. In chapter 5, we prove a conditional variational principle for topological pressure for non-compact sets of the form

$$\left\{x \in X : \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \varphi(f^i(x)) = \alpha \right\},$$

generalising a previously known result for topological entropy. As one application, we prove multifractal analysis results for the entropy spectrum of a suspension flow over a continuous map with specification.

In chapter 6, we assume that $f : X \mapsto X$ is a continuous map satisfying a property we call almost specification (which is weaker than specification). We show that the set of points for which the Birkhoff average of φ does not exist is either empty or has full topological entropy. Every β-shift satisfies almost specification and we show that the irregular set for any β-shift or β-transformation is either empty or has full topological entropy and Hausdorff dimension.

In chapter 7, we introduce an alternative definition of topological pressure for arbitrary (non-compact, non-invariant) Borel subsets of metric spaces. This new quantity is defined via a suitable conditional variational principle, leading to an alternative definition of an equilibrium state. We study the properties of this new quantity and compare it with existing notions of topological pressure. We apply our new definition to some interesting examples, including the level sets of the pointwise Lyapunov exponent for the Manneville-Pomeau family of maps.