# Practice problems – Exam 1

P1. Can you do your old homework problems if you scramble the problems up and present them out of context (i.e. if you don't know what section the problems came from)?

P2. All the problems from section 2.3 are good practice.

P3. (a) Suppose $M(x,y)+N(x,y)\frac{dy}{dx}=0$ is not exact. Derive a criterion on $M$ and $N$ which allows for an integrating factor $\mu(y)$ which depends upon $y$ alone, which makes $\mu M+\mu N\frac{dy}{dx}=0$ exact.

(b) Consider the differential equation $\frac{x}{y}+\frac{dy}{dx}=0$. Use your results from (a) to derive an integrating factor $\mu(y)$ which makes the equation exact.

P4. Glucose is being fed into a patient's bloodstream at a rate of $c$ grams per minute. The patient's body removes glucose from the bloodstream at a rate proportional to the amount of glucose present.

(a) Write a differential equation modeling the amount of glucose in the patient's bloodstream.

(b) What is the amount of glucose in the bloodstream in the limit as $t\to\infty$?

P5. Consider the differential equation $y'=y^2$. What is wrong with the following two statements:

(a) "$y'=f(y)$ with $f(y)=y^2$. As $f$ and $\frac{\partial f}{\partial y}$ are continuous for all $y$, a unique solution exists for all time."

(b) "The coefficient of the $y$ term is continuous for all $t$ in $(-\infty,\infty)$, so a unique solution exists for all $t$."

(c) Give a correct statement regarding existence and uniqueness of solutions for this differential equation.