Math 2586 sample Final

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Show your work!

Problems

1. Give the detailed definition of vector spaces.
2. Consider the following 3×3 matrix
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$
Find an invertible 3×3 matrix S and a diagonal 3×3 matrix D such that $AS = SD$.
3. Let A be a 3×3 matrix with 3 distinct eigenvalues. Denote by
$$c_0 + c_1 x + c_2 x^2 + c_3 x^3$$
the characteristic polynomial of A. Show that
$$c_0 I_3 + c_1 A + c_2 A^2 + c_3 A^3 = O.$$
4. Determine whether the following matrix is invertible or not:
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$
If A is invertible, compute A^{-1}.
5. Show that the set of polynomials in x of degree at most 5 is a vector space.
6. Give the definition of orthogonal matrices.