

Math 2586 sample midterm 2

Hsian-Hua Tseng

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Show your work!

Problems

1. Give the detailed definition of vector spaces.
2. Consider the following matrix:

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & -3 & 5 \\ 1 & 0 & 7 \end{pmatrix}.$$

Find bases for the following subspaces:

- (a) $\mathcal{N}(A)$, the null space of A .
- (b) $\mathcal{R}(A)$, the range of A .

3. Consider the following vectors in \mathbb{R}^3 :

$$v_1 = (3, 5, 0), \quad v_2 = (6, 4, 2).$$

Find a vector $u \in \mathbb{R}^3$ such that

- (a) The length of u is 1.
- (b) u is orthogonal to both v_1 and v_2 .

4. Determine whether the following matrix is invertible or not:

$$A = \begin{pmatrix} -1 & -2 & 11 \\ 1 & 3 & -15 \\ 0 & -1 & 5 \end{pmatrix}.$$

If A is invertible, compute A^{-1} .

5. Determine the dimension of the row space of the following matrix:

$$A = \begin{pmatrix} 1 & -2 & -3 \\ -4 & 5 & -6 \\ -9 & -8 & 7 \\ 10 & -11 & 12 \end{pmatrix}.$$