We will be going over some of the problems on Monday, 2/25. You may prepare one to talk about for a few bonus points; please email me if you want to ensure a chance to speak.

1. **Disprove.** Let $P(x)$ and $Q(x)$ be unary predicate symbols. Then
   
   $$\forall x (P(x) \lor Q(x)) \equiv \forall x P(x) \lor \forall y Q(y)$$

2. Let $P$ and $Q$ be propositional forms, neither of which are tautologies.
   
   (a) **Prove or Disprove.** The propositional form $P \lor Q$ is not a tautology.
   
   (b) **Prove or Disprove.** The propositional form $P \land Q$ is not a tautology.

3. **Prove.** Let $x$ be an integer and suppose that there is a rational number $c$ such that $c^4 = x$. Then $c$ is an integer.

4. **Prove or Disprove.** Let $a$, $b$, and $c$ be positive integers such that $a$ divides $bc$. Then $a$ divides $b$ or $a$ divides $c$.

5. **Prove by induction.** Let $x$ be a nonnegative integer. Then there is an $r \in \{0, 1\}$ such that $x \equiv r \mod 2$.

6. **Prove.** For any positive integer $n$, the sum $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

7. **Prove.** For any positive integer $n$, $3$ divides $4^n - 1$.

8. **Prove.** For any nonnegative integer $n$,
   
   $$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$