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Math 3345

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Midterm 1

We will be going over some of the problems on Monday, 2/25. You may prepare one to talk about for a few bonus points; please email me if you want to ensure a chance to speak.

1. **Disprove.** Let $P(x)$ and $Q(x)$ be unary predicate symbols. Then

$$\forall x(P(x) \vee Q(x)) \equiv \forall x P(x) \vee \forall y Q(y)$$

2. Let P and Q be propositional forms, neither of which are tautologies.

   (a) **Prove or Disprove.** The propositional form $P \vee Q$ is not a tautology.

   (b) **Prove or Disprove.** The propositional form $P \wedge Q$ is not a tautology.

3. **Prove.** Let x be an integer and suppose that there is a rational number c such that $c^4 = x$. Then c is an integer.

4. **Prove or Disprove.** Let a, b, and c be positive integers such that a divides bc. Then a divides b or a divides c.

5. **Prove by induction.** Let x be a nonnegative integer. Then there is an $r \in \{0, 1\}$ such that $x \equiv r \mod 2$.

6. **Prove.** For any positive integer n, the sum $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

7. **Prove.** For any positive integer n, 3 divides $4^n - 1$.

8. **Prove.** For any nonnegative integer n,

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$