Midterm # 2

1. **Prove or Disprove.** Let A, B, and C be nonempty sets and let $f : A \to B$ and $g : B \to C$ both be bijections onto their respective targets. Let $h = g \circ f$. Then h is a bijection from A to C and $h^{-1} = f^{-1} \circ g^{-1}$.

2. **Prove or Disprove.** Let A, B, and C be sets such that $C \subseteq B \subseteq A$. Then $(A \setminus C) \setminus (B \setminus C) = A \setminus B$.

3. **Prove or Disprove.** Let S be a set and let $\mathcal{M}$ be a collection of subsets of S. Let $A \subseteq S$. Then $A \cap \bigcup \mathcal{M} = \bigcup \{M \cap A : M \in \mathcal{M}\}$.

4. **Prove or Disprove.** Let A, B, S, and T be sets such that

   - A is equinumerous to S,
   - B is equinumerous to T, and
   - $A \cap B = \emptyset = S \cap T$.

   Then $A \cup B$ is equinumerous to $S \cup T$.

5. **Prove or Disprove.** Let A, B, S, and T be sets such that

   - A is equinumerous to B,
   - S is equinumerous to T,
   - $A \subseteq S$, and
   - $B \subseteq T$.

   Then $S \setminus A$ is equinumerous to $T \setminus B$.

6. **Prove.** Let A and B be nonempty sets, and suppose $f : A \to B$. Then function f is an surjection from A to B if and only if for every $T \subseteq B$ we have $f[f^{-1}[T]] = T$.

7. **Prove *directly* by induction.** Let $n \in \mathbb{N}$ be such that $n > 1$. Then there is a prime number $p \in \mathbb{N}$ such that p divides n.

8. **Prove or Disprove.** Let A and B be sets and let $f : A \to B$ be a surjection from A to B. Then there is a function $g : B \to A$ such that $f \circ g = id_B$.