January 23, 2013

Math 3345

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Quiz # 1

Name: ....

Class time: ....

Respond to *exactly two* of the following four problems on the blank paper given. Make sure that your name is on each sheet that you turn in, as well as a page number if you require multiple pages for a given problem. Each problem must be on a separate sheet; turn in each sheet to the appropriate pile. For each problem that involves writing a proof, you are to explicitly formulate a claim and then give a proof. Please read each problem carefully before attempting it. You must turn in this sheet and all scratch paper, but nothing will be graded unless you indicate to do so (*i.e.* you may use this sheet as scratch paper or to write a solution to one problem). Each of the problems is given equal weight in grading. Attempts at more than two problems will result in a random selection of two of your solutions being graded.

1. **Prove or Disprove.** Let P and Q be propositional forms such that both P and $P \to Q$ are tautologies. Then Q is a tautology.

2. **Prove or Disprove.** For propositional forms P and Q, the connective $P \downarrow Q$ is given by the truth table:

P	Q	$P \downarrow Q$
T	T	F
T	F	F
F	T	F
F	F	T

   There is a propositional form involving no connectives other than $\neg$ and $\wedge$ that is logically equivalent to $P \downarrow Q$.

3. A set of real numbers A is **bounded above** if there is a real number larger than every element of A. Write out this definition formally using the predicate $<$, and then briefly explain why your formulation is equivalent to the English version of the definition given above.

4. **Disprove.** Let $P(x)$ and $Q(x)$ be unary predicate symbols. Then

$$\exists x(P(x) \wedge Q(x)) \equiv \exists x P(x) \wedge \exists y Q(y)$$