Respond to exactly two of the following four problems on the blank paper given. Make sure that your name is on each sheet that you turn in, as well as a page number if you require multiple pages for a given problem. Each problem must be on a separate sheet; turn in each sheet to the appropriate pile. For each problem that involves writing a proof, you are to explicitly formulate a claim and then give a proof. Please read each problem carefully before attempting it. You must turn in this sheet and all scratch paper, but nothing will be graded unless you indicate to do so (i.e. you may use this sheet as scratch paper or to write a solution to one problem). Each of the problems is given equal weight in grading. Attempts at more than two problems will result in a random selection of two of your solutions being graded.

1. **Prove or Disprove.** Let $a$ and $b$ be positive integers and suppose that for every positive integer $c$ that $a \equiv b \pmod{c}$. Then $a = b$.

2. **Prove or Disprove.** Suppose that $x, y \in \mathbb{R}$ are such that both $x + y$ and $xy$ are positive rational numbers. Then $x$ and $y$ are both rational numbers.

3. **Prove or Disprove.** For any integer $x$, the number $x^2 + x + 1$ is odd. (*You may assume that every integer is either even or odd.*)

4. **Definition.** Give the precise definition of the symbol $k | n$. Be sure to declare the variables $k$ and $n$. 