March 27, 2013

Math 3345

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Quiz # 3

Name: ....

Class time: ....

Respond to *exactly two* of the following four problems on the blank paper given. Make sure that your name is on each sheet that you turn in, as well as a page number if you require multiple pages for a given problem. Each problem must be on a separate sheet; turn in each sheet to the appropriate pile. For each problem that involves writing a proof, you are to explicitly formulate a claim and then give a proof. Please read each problem carefully before attempting it. You must turn in this sheet and all scratch paper, but nothing will be graded unless you indicate to do so (*i.e.* you may use this sheet as scratch paper or to write a solution to one problem). Each of the problems is given equal weight in grading. Attempts at more than two problems will result in a random selection of two of your solutions being graded.

1. **Prove or Disprove.** Let A and B be sets and let $f : A \to B$. If f has an inverse function $g : B \to A$, then this inverse is unique.

2. Let $\mathcal{C} \subseteq \mathcal{P}(\mathbb{R})$ be the collection of sets $\mathcal{C} = \{A \subseteq \mathbb{R} : A \text{ is either an open interval or } A = \emptyset\}$.

   (a) **Prove or Disprove.** For any subcollection $\mathcal{D} \subseteq \mathcal{C}$, we have that $\cap\mathcal{D} \in \mathcal{C}$.

   (b) **Prove or Disprove.** For any subcollection $\mathcal{D} \subseteq \mathcal{C}$, we have that $\cup\mathcal{D} \in \mathcal{C}$.

3. **Prove or Disprove.** For any set A, we have that $A = \emptyset$ if and only if for every set B we have that $A \subseteq B$.

4. **Prove or Disprove.** Let A, B, C, and D be sets such that A and B are disjoint, and let $f : A \to C$ and $g : B \to D$ both be injective functions. Let $X = A \cup B$ and let $Y = C \cup D$. Define h by

$$h(x) := \begin{cases} f(x), & \text{if } x \in A \\ g(x), & \text{if } x \in B. \end{cases}$$

   Then $h : X \to Y$ is a function and h is injective.