Math 8440—Homework 12, due November 13

(1) Prove that $SL_2(\mathbb{R}) = KAN^+$ where

$$K = \left\{ \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix} : t \in \mathbb{R} \right\}.$$

(Hint: this may be easiest to prove if you make use of the connection between $PSL_2(\mathbb{R})$ and $T^1H$.)

(2) Prove the Gauss-Bonnet theorem for hyperbolic triangles with a point at infinity. That is, calculate the area of a hyperbolic region bounded by two vertical geodesics and one non-vertical geodesic (all of which touch) and show the area is equal to $\pi$ minus the sum of the interior angles (measured by the angle between the tangent vectors).

(3) Prove the Gauss-Bonnet theorem for all hyperbolic triangles. That is, if you have three geodesics which intersect pairwise, then the bounded region has area equal to $\pi$ minus the sum of the interior angles.

(4) Prove that a Dirichlet region is a fundamental region.

(5) Let $p, p' \in \mathbb{H}$ be distinct points. Prove that the set

$$\{ z \in \mathbb{H} : d(z, p) = d(z, p') \}$$

is a geodesic. (Hint: find a single pair $p, p'$ where this is easy to prove and extend from there.)

(6) Prove that the region

$$\{ z \in \mathbb{H} : |z| \geq 1, |\Re(z)| \leq 1/2 \}$$

is a Dirichlet region corresponding to the point $2i$. 