List of groups

This is a list of groups which will be encountered in the course. Probably you will have met them already. There will be more detail when we come to make use of them.

Cyclic groups

The cyclic group of order n, denoted C_n, is often written as

$$\mathbb{Z}/n\mathbb{Z} = \{0, 1, 2, \ldots, n-1\}$$

under addition mod n.

Equally often, C_n is taken to be the set of nth roots of 1 in the complex plane, under multiplication.

Symmetric groups

The symmetric group on n symbols, denoted S_n, is the group of all permutations on $\{1, 2, \ldots, n\}$ or on any other convenient set of n symbols. Note: we compose permutations from right to left as with maps in general.

Alternating groups

The alternating group on n symbols, denoted A_n, is the group of all permutations on $\{1, 2, \ldots, n\}$ (or on any other convenient set of n symbols) which have even parity.

Remember that any permutation $\sigma \in S_n$ can be written as a product of transpositions, and that the parity (even/odd) of the number of transpositions is the same for all such products.

Dihedral groups

The dihedral groups are the isometry groups of regular polygons in the plane.

We will denote the dihedral group for the n-gon by D_n. It is also sometimes denoted D_{2n}, since it has $2n$ elements.

Denote rotation through $\frac{2\pi}{n}$ by R. Then R generates a cyclic subgroup, $\{I, R, R^2, \ldots, R^{n-1}\}$. Denote reflection by F; clearly $F^2 = I$. Lastly, $FRF = R^{-1}$.